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RADIATION PROJECT INERNAL REPORT NO. 13

ELECTRON MOTION IN COMBINED H_z, H_ϕ

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We have
$$\ddot{x} = \omega_z \dot{y} - \omega_y \dot{z}$$
$$\ddot{y} = \omega_x \dot{z} - \omega_z \dot{x} ,$$

where $\underline{\omega} \equiv e\underline{H}/\gamma mc$. Also $\omega_{x,y} = \omega_\phi(-y,x)/R$, so that
$$\ddot{x} = \omega_z \dot{y} + \omega_\phi \dot{z} x/R$$
$$\ddot{y} = -\omega_\phi \dot{z} y/R - \omega_z \dot{x} .$$

For the axial motion
$$\ddot{z} = \omega_\phi (x\dot{x}/R + y\dot{y}/R) = \omega_\phi dR/dt.$$

We take ω_z = const.; $\theta \equiv \omega_z t$, $d/d\theta \equiv {}'$. Thus
$$x'' = y' - Az'x/R^2$$
$$y'' = -Az'y/R^2 - x'$$
$$z'' = (A/R)R',$$

where $A \equiv R\omega_\phi/\omega_z = 2i/H_z$ if ω_ϕ is produced by a central current i(emu). These nonlinear equations are fairly readily solved numerically. Usually the electron can be taken as starting from the point $(x_o,0,0)$. The initial velocity is in these units $v/\omega_z = \frac{v \cdot \gamma mc}{eH_z} = \frac{1700\beta\gamma}{H_z}$.

Under conditions of normal emission this value is used for z_o'.

It is seen from the numerical solutions that in many cases R, and so ω_ϕ, is about constant. From the equations for $\ddot{x}$ and $\ddot{y}$ one can form the single equation

$$\ddot{\xi} + i\omega_z\dot{\xi} + (\omega_\phi \dot{z}/R)\,\xi = 0$$

on defining $\xi \equiv x + iy$. If the coefficients (including $\dot{z}$) are effectively constant, then this equation admits of the solution $\xi = \xi_o e^{\lambda t}$, where

$$\lambda^2 + i\omega_z\lambda + \omega_\phi \dot{z}/R = 0,$$

or

$$\lambda = -\tfrac{1}{2}\,i\omega_z \pm i\sqrt{\tfrac{1}{4}\,\omega_z^{\,2} + \omega_\phi \dot{z}/R}\quad .$$

In the usual pinching mode $\omega_z = 0$, $\omega_\phi > 0$; obviously an R=constant approximation is inadequate for this case. As $w \equiv \frac{1}{2}\,\omega_z\sqrt{\omega_\phi \dot{z}/R}$ increases away from zero, two distinct roots appear:

w	$\lambda_1/i\sqrt{\omega_\phi \dot{z}/R}$	$\lambda_2/i\sqrt{\omega_\phi \dot{z}/R}$
0	1	-1
1	0.414	-2.414
2	0.236	-4.236
3	0.162	-6.162
4	0.123	-8.123
5	0.099	-10.099

We can represent the motion as $\xi = \xi_1 e^{\lambda_1 t} + \xi_2{}^{\lambda_2 t}$, $\dot{\xi} = \lambda_1 \xi_1 e^{\lambda_1 t} + \lambda_2 \xi_2 e^{\lambda_2 t}$. From the "preferred" starting condition $\xi(0) = x_o + i0$, $\xi(0) = 0$, we have

$$\xi_1 + \xi_2 = x_o$$

$$\lambda_1 \xi_1 + \lambda_2 \xi_2 = 0$$

so that

$$\xi = x_o(1-\lambda_1/\lambda_2)^{-1}\,[e^{\lambda_1 t} - (\lambda_1/\lambda_2)e^{\lambda_2 t}]\ .$$

Thus the motion involves a superposition of rotating vectors in which the amplitudes are inverse with the frequencies. For example at $w = 1$, $|\lambda_1/\lambda_2| = \frac{\sqrt{2}-1}{\sqrt{2}+1} = 0.172$, and one would expect the ratio of minimum to maximum amplitudes, corresponding to the rotating vectors being out of and in phase, to be $1/\sqrt{2} = 0.707$, or in general

$$\frac{R_{min}}{R_{max}} = \frac{w}{\sqrt{w^2+1}}\ .$$

Any value of ω greater than ~ 1 is thus adequate to suppress strongly the tendency to pinch.

We have

$$w = \left(\frac{\omega_z}{\omega_\phi}\right)\left(\frac{\omega_\phi}{4\dot{z}/R}\right)^{\frac{1}{2}}$$

and

$$\omega_\phi = (e/\gamma mc)(2i/R) = 2(\nu/\gamma)(\dot{z}/R).$$

Thus

$$w = \left(\frac{\omega_z}{\omega_\phi}\right)\left(2\,\frac{\nu}{\gamma}\right)^{\frac{1}{2}}\ .$$

Thus as ν/γ increases, the ratio of ω_z to ω_ϕ can be reduced if ω is kept constant.

Some numerical results (Fig. 1) have been obtained for $z_o' = 0.2$, $A = 4$ and 2, with $x_o = 1$ cm, $y_o = z_o = x_o' = y_o' = 0$.

If $\beta\gamma$ is taken as $\sqrt{3}$, corresponding to 510-keV electrons, then these figures correspond to H_z = 15 kG, i = 300 and 150 kA, respectively. It is seen from the numerical results that the mean value of z′ is considerably below the starting value. We have in general $\nu = \frac{i_{amp}}{17000\bar{\beta}}$, $H_\phi = 0.2 i_{amp}$ since R ~ 1 cm. Thus for the two cases, for γ=2, $\omega = 1.03\bar{\beta}^{-\frac{1}{2}}$ and $1.46\bar{\beta}^{-\frac{1}{2}}$.

We infer the numerical values of w from the ratio R_{min}/R_{max} as found from the numerical solution. For the two cases w_{num} = 1.43 and 1.17. From these one can in turn get $\bar{\beta}$ as 0.519 and 0.642. The value of β corresponding to the initial forward velocity is 0.866 (510-keV electrons), but the mean values inferred from the z-solutions obtained from the program are very much less (0.104 and 0.303). It is therefore difficult to obtain numerically useful results from the linearized solutions for axial fields as low as these. It is interesting that the numerical solutions actually show a higher value of w for the higher current, presumably because of the lower effective value of $\bar{\beta}$. The linearized solutions with the initial value of β used for $\bar{\beta}$ are very crude and can only represent a rough guide.

The question of what meaning these results have in a diode, where electric fields, ions, etc., are individually or collectively present, is relevant. If the tube functions as a vacuum device, the electric fields are significant and modify the orbits, but it may be recalled that the orbit description corresponding to constant energy in a magnetic field alone differs little from one in which the electron gains (or loses) energy as it moves. If the tube is partially filled with plasma proceeding backward from anode to cathode, and if only electric neutralization is involved, the pure-magnetic-field description is probably satisfactory; magnetic neutralization under these circumstances is somewhat hard to envision*.

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*Though certain computer results seem to reveal the existence of counter-streaming currents along the axis even in the vacuum mode.

Radial electric fields are supposed small due to electrode proximity.

The remarkably low magnetic field values sufficient to suppress pinching in this flow observed on Gamble-I, and which were certainly a surprise to this writer, augur well for beam control by this technique.

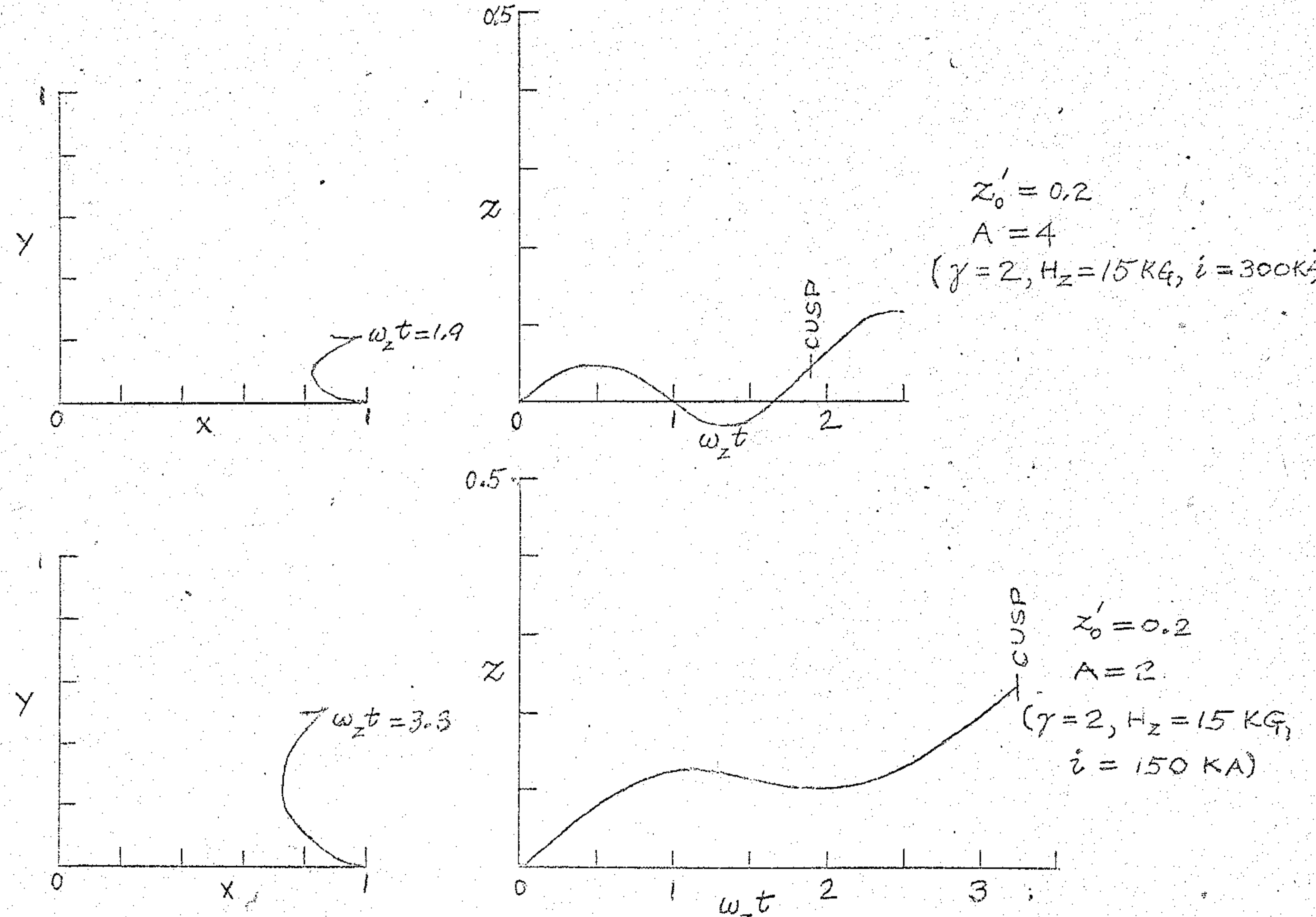

FIGURE 1: REPRESENTATIVE TRAJECTORIES IN COMBINED AXIAL AND AZIMUTHAL MAGNETIC FIELDS